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George C. Marshall Space Flight Center
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Attention: Purchasing Office PR-EC

Subject: Quarterly Report No. 7 - Contract NAS8-11341

Gentlemen:

This study was conducted to determine if significant forces could be generated by the movement of one servoactuator, transmitted through the mechanical linkage to the piston of the other servoactuator and thereby create a pressure surge of sufficient magnitude to cause structural failure of hydraulic components.

Introduction

In the analysis, certain simplifying assumptions were made. These will be enumerated when used. The assumptions are always chosen so that the effect will be conservative, i.e., worst case conditions are established. One of the first is that the spring constant is ∞, i.e., no energy may be stored in the mechanical configuration. A point mass is also assumed and the equation of motion for this mass is derived. The motion is due to the extension or retraction of one servoactuator piston (henceforth called servoactuator No. 1) while maintaining fixed lengths from the point mass position to the engine gimbal point and to the vehicle tie-point of the other servoactuator (henceforth called servoactuator No. 2).

Geometrical Configuration of System

The engineering data used to establish coordinate systems were obtained from Figure 2-15A in Section II of report R-3896-1 and from the drawing of the F-1 Engine Simulator Stand prepared by the Brown Engineering Company and dated 6-27-65.

The first approach was evolved in the following manner. Since the length from the point mass to the gimbal point is assumed to remain fixed, motion of the point mass about the gimbal point describes a sphere. In like manner the motion of the point mass about the fixed length servoactuator No. 2 vehicle tie-point also describes a sphere. The intersection of these two spheres results in a circle which is the trajectory of the motion of the point mass.

The first coordinate transformation is made to orient the coordinate system so that the plane containing the circular trajectory is parallel to one of the planes formed by two coordinate axes resulting in a simpler expression for the motion of the point mass.

The second coordinate transformation is made to translate the origin of the coordinate system to the neutral position of the point mass so that the coordinates of the point mass position vector are all zero when the engine is in the neutral position.

The different coordinate systems and identification of nomenclature are given in Figure 1.

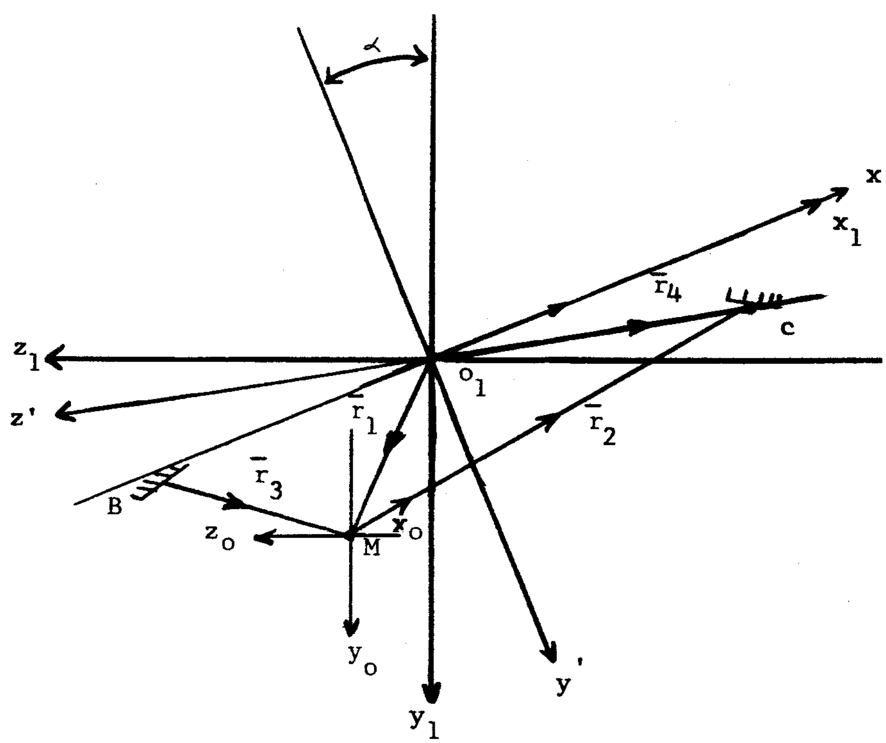

Figure 1: - Location of Equivalent Point Mass of the Engine with Respect to Three Cartesian Coordinate Systems.

Where:

(1) O_1 - the gimbal point of the engine.

(2) M - the equivalent point mass of the engine.

(3) C - the fixed point in the y_1, z_1 plane to which actuator No. 2 is attached.

(4) B - the fixed point in the x_1, y_1 plane to which actuator No. 1 is attached.

(5) the x_1, y_1, z_1 coordinate system corresponds to the x_g', y, z_g' coordinate system in Figure 2-15A from R-3896-1 Report, Section II.

(6) $\bar{r}_1$ - the magnitude $|\bar{r}_1|$ is the length of the vector from the gimbal point of the engine to the point mass of the engine.

(7) $\bar{r}_2$ - the magnitude $|\bar{r}_2|$ is the equivalent fixed length of servoactuator No. 2, i.e., the length from the servoactuator vehicle tie-point to the point mass of the engine.

(8) $\bar{r}_3$ - the magnitude $|\bar{r}_3|$ is the equivalent variable length of servoactuator No. 1, i.e., the length from the servoactuator vehicle tie-point to the point mass of the engine.

(9) $\bar{r}_4$ - the magnitude $|\bar{r}_4|$ is the length of the vector from the gimbal point to the vehicle tie-point of servoactuator No. 2.

(10) α - angle through which x_1, y_1, z_1 coordinate system is rotated about the x_1 axis to establish the x', y', z' coordinate system. α = constant = 8°.

First Approach

In the x_1, y_1, z_1 coordinate system with its origin at the gimbal point 0_1 the fixed vector $\bar{r}_4$ and the neutral positions of the vectors $\bar{r}_1$ and $\bar{r}_2$ are given by:[1,2]

$$\begin{aligned} \bar{r}_1 &= (12.8)\,\bar{\mu}_{x_1} + (52.8)\,\bar{\mu}_{y_1} + (12.7)\,\bar{\mu}_{z_1} \\ \bar{r}_4 &= (-10.5)\,\bar{\mu}_{y_1} + (-75)\,\bar{\mu}_{z_1} \\ \bar{r}_2 &= \bar{r}_1 - \bar{r}_4 = (12.8)\,\bar{\mu}_{x_1} + (63.3)\,\bar{\mu}_{y_1} + (87.7)\,\bar{\mu}_{z_1} \end{aligned} \tag{1}$$

where $\bar{\mu}_{x_1}$, $\bar{\mu}_{y_1}$ and $\bar{\mu}_{z_1}$ are unit vectors directed along the positive x_1, y_1, z_1 axes respectively.

Now since it is assumed that the lengths of the vectors $\bar{r}_1$ and $\bar{r}_2$ do not change upon motion of the point mass then

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(1) Superscripts refer to reference number

$$\left|\bar{r}_1\right| = \text{constant} = 55.8'',$$

and
$$\left|\bar{r}_2\right| = \text{constant} = 108.9'' \qquad (2)$$
$$\left|\bar{r}_4\right| = \text{constant} = 75.729''.$$

In the x', y', z' coordinate systems with its origin at the gimbal point O_1, the trajectory of the motion of the point mass describes a circle which is contained in a plane parallel to the plane formed by the x', y' coordinate axes. This x', y', z' coordinate system is obtained from the x_1, y_1, z_1 coordinate system by a rotation. The x_1, y_1, z_1 coordinate system is rotated about the x axis until the z' axis (corresponds to the z_1 axis before rotation) passes through the vehicle tie-point of servoactuator No. 2. This will then give the geometrical relationships depicted in Figure 2. The transformation equations will be given directly.

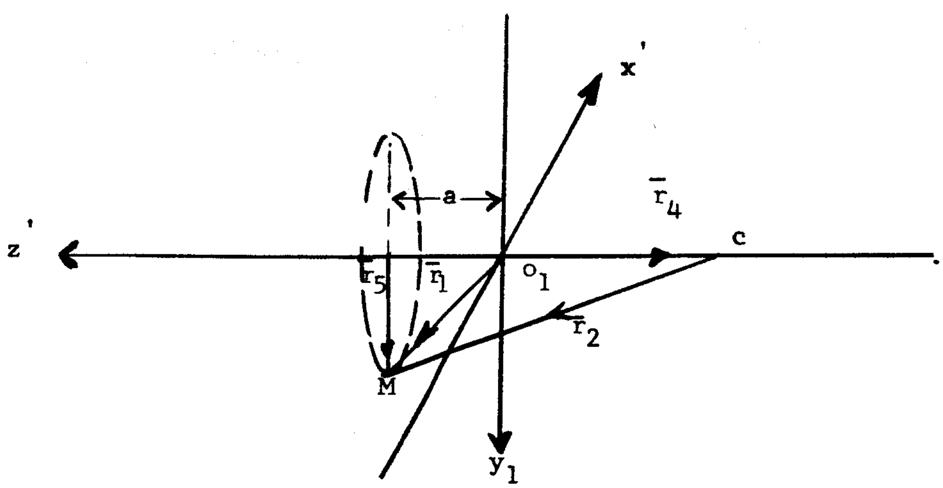


Figure 2: - Path Described by Equivalent Point Mass of Engine in x', y', z' Coordinate System.

With respect to the x', y', z' coordinate system the path described by the point mass (M) of the engine, with the constraints that $\left|\bar{r}_1\right|$ = constant and $\left|\bar{r}_2\right|$ = constant, is a circle parallel to the x', y' plane. The center of the circle is displaced a constant distance, "a", on the z' axis.

At $z' = a$ the equation of the circle is:

$$x'^2 + y'^2 = \left|\bar{r}_5\right|^2 \tag{3}$$

therefore:

$$y' = \pm\sqrt{\left|\bar{r}_5\right|^2 - x'^2}\,, \text{ where } \left|\bar{r}_5\right| = \text{constant.}$$
$$= \text{radius of circle} \tag{4}$$

The vector $\bar{r}_1$ can be expressed as:

$$\bar{r}_1 = (x')\,\bar{\mu}_{x'} + (y')\,\bar{\mu}_{y'} + (a)\,\bar{\mu}_{z'}$$

where $\bar{\mu}_{x'}$, $\bar{\mu}_{y'}$ and $\bar{\mu}_{z'}$ are unit vectors directed along the positive x', y' and z' axes respectively. Using Equation (4)

$$\bar{r}_1 = (x')\,\bar{\mu}_{x'} + \left[\sqrt{\left|\bar{r}_5\right|^2 - (x')^2}\right]\bar{\mu}_{y'} + (a)\,\bar{\mu}_{z'}\ . \tag{5}$$

It should be pointed out that for practical displacements (physical limitations on amount of retraction or extension of servoactuator pistons) of the point mass the values of the coordinate y' will be positive so that only the positive solution given by Equation (4) need be retained and such is the case in Equation (5).

The value of the length a can be determined from Figure 3 and Equation (2).

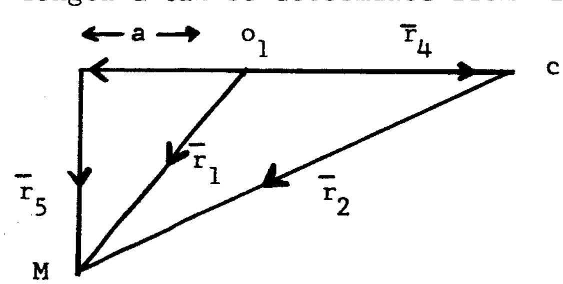


Figure 3: - Figure Used to Determine the Value of (a).

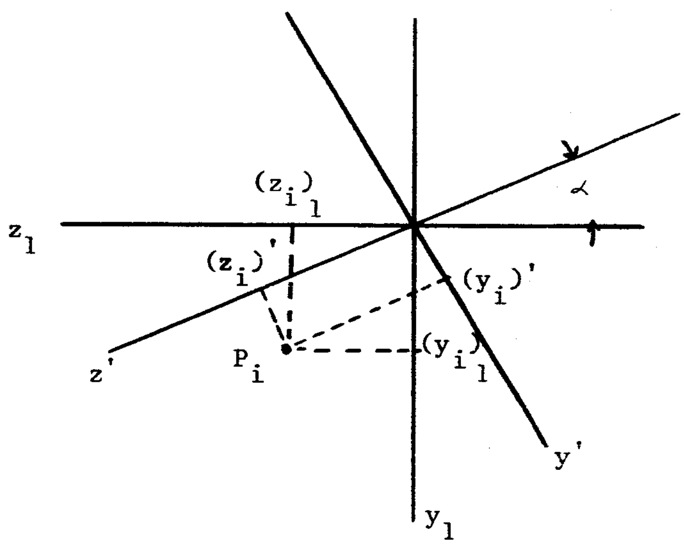
$$a = \frac{\left|\bar{r}_2\right|^2 - \left|\bar{r}_1\right|^2 - \left|\bar{r}_4\right|^2}{2\left|\bar{r}_4\right|} \approx 20'' \tag{6}$$

and

$$\left|\bar{r}_5\right| \approx \sqrt{\left|\bar{r}_1\right|^2 - a^2} \approx 52.2'' \tag{7}$$

It is now desired to write the vector $\bar{r}_1$ in the x_o, y_o, z_o coordinate system. This is a coordinate system whose origin is at the neutral position of the point mass and is accomplished by a translation of the original x_1, y_1, z_1 coordinate system. Transforming the vector $\bar{r}_1$ given by Equation 5 to the x_1, y_1, z_1 coordinate system results in: (See Figure 4)

$$\bar{r}_1 = \bar{\mu}_{x_1} x_1 + \bar{\mu}_{y_1} \left(\sqrt{\bar{r}_5^{\,2} - x_1^2}\cos\alpha + a \sin\alpha\right) + \bar{\mu}_{z_1}\left(a\cos\alpha - \sqrt{r_5^2 - x_1^2}\sin\alpha\right) \tag{8}$$



$\alpha = \tan^{-1} \dfrac{10.5}{75} = 8°$

Coordinates rotated about x_1 axis

Figure 4: - Transformation of x_1, y_1, z_1 Coordinate System into x', y', z' Coordinate System

<u>Coordinate Transformation</u> or <u>Unit Vector Transformations</u>

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \bullet \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \qquad \begin{bmatrix} \bar{\mu}_{x'} \\ \bar{\mu}_{y'} \\ \bar{\mu}_{z'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \bullet \begin{bmatrix} \bar{\mu}_{x_1} \\ \bar{\mu}_{y_1} \\ \bar{\mu}_{z_1} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \bullet \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \qquad \begin{bmatrix} \bar{\mu}_{x_1} \\ \bar{\mu}_{y_1} \\ \bar{\mu}_{z_1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} \bullet \begin{bmatrix} \bar{\mu}_{x'} \\ \bar{\mu}_{y'} \\ \bar{\mu}_{z'} \end{bmatrix}$$

The vector $\bar{r}_1$ can now be written in the x_o, y_o, z_o coordinate system which has the neutral position of the point mass as its origin. The transformation between the x_1, y_1, z_1 coordinate system and the x_o, y_o, z_o coordinate system is a linear displacement and the transformations are given in Equation (9).

$$\begin{array}{lll} \underline{\text{Coordinates}} & \underline{\text{Unit Vectors}} & \\ x_1 = x_o + k_1\,, & \bar{\mu}_{x_o} = \bar{\mu}_{x_1} & \text{and} \quad k_1 = 12.8 \\ y_1 = y_o + k_2\,, & \bar{\mu}_{y_o} = \bar{\mu}_{y_1} & \text{and} \quad k_2 = 52.8 \\ z_1 = z_o + k_3\,, & \bar{\mu}_{z_o} = \bar{\mu}_{z_1} & \text{and} \quad k_3 = 12.7 \end{array} \qquad (9)$$

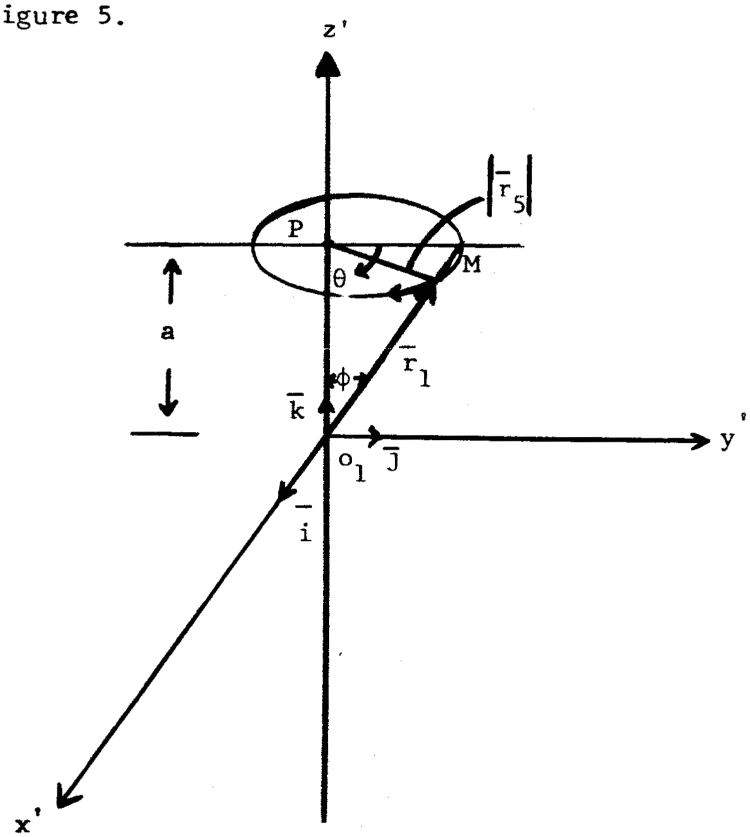
The vector $\bar{r}_1$ is now written as:

$$\bar{r}_1 = \bar{\mu}_{x_o}\,(x_o + k_1) + \bar{\mu}_{y_o}\left(\sqrt{r_5^2 - (x_o + k_1)^2}\,\cos\alpha + a\sin\alpha + k_2\right) + \bar{\mu}_{z_o}\left(a\cos\alpha - \sqrt{r_5^2 - (x_o + k_1)^2}\,\sin\alpha + k_3\right) \qquad (10)$$

Equation (10) is an exact expression for the vector $\bar{r}_1$ and is a function of only one variable (x_o). The position of the equivalent point mass (M) of the engine can be determined by the vector $\bar{r}_1$.

A solution for the equation of motion of the equivalent point mass of the engine as a function of time (assuming a given extension of the actuator as a function of time) was attempted using Equation (10). The complexity of the equations obtained by using this approach became unwieldy for hand computation and another approach was sought. It should be pointed out that the possibility of machine computation was kept in mind and if results from the succeeding approximation indicated the generation of detrimental pressure surges, the exact solution would be sought.

An approximate solution will now be given. Redrawing Figure 2 in a manner more conducive to analysis by angular coordinates results in Figure 5.



θ measured from plane containing z' axis and point of neutral position of point mass

$$\theta = \sin^{-1} \frac{|\bar{r}_5|}{|\bar{r}_1|} = \text{constant.}$$

<u>Simplified Unit Vector Notation</u>

$\bar{i} = \bar{\mu}_{x'}$

$\bar{j} = \bar{\mu}_{y'}$

$\bar{k} = \bar{\mu}_{z'}$

Figure 5: - Path Described by Equivalent Point Mass in x', y', z' Coordinate System.

A given extension in actuactor No. 1 will cause a displacement of the point mass in the + θ direction as shown in Figure 6.

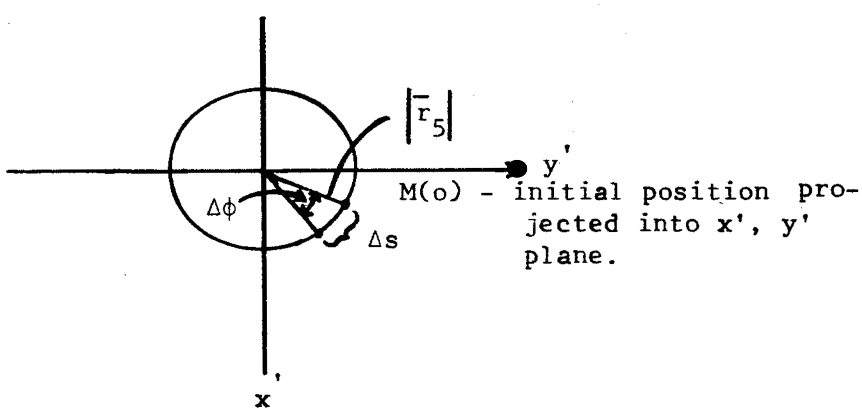


Figure 6: - Projected Path in x', y' Plane.

From Figure 6:

$$\Delta s = \left|\bar{r}_5\right| \Delta\theta = \left(\left|\bar{r}_1\right| \sin \phi\right) \Delta\theta \tag{11}$$

The tangential speed of point M is:

$$v = \frac{ds}{dt} = \left|\bar{r}_1\right| \dot{\theta} \sin \phi \tag{12}$$

Where $\dot{\theta}$ = angular speed of the point M

Using vector notation[3], the angular velocity is given by

$$\bar{\omega} = -\dot{\theta}\ \bar{k}, \tag{13}$$

and the velocity of the point M is

$$\bar{v} = \frac{d\bar{r}_1}{dt} = \bar{\omega} \times \bar{r}_1 . \tag{14}$$

The acceleration of point M is

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}\left(\bar{\omega} \times \bar{r}_1\right) = \frac{d\bar{\omega}}{dt} \times \bar{r}_1 + \bar{\omega} \times \frac{d\bar{r}_1}{dt} . \tag{15}$$

Rewriting Equation (15):

$$\bar{a} = \bar{\alpha} \times \bar{r}_1 + \bar{\omega} \times \left(\bar{\omega} \times \bar{r}_1\right) \tag{16}$$

Where $\bar{\alpha} = -\alpha\ \bar{k} = -\dot{\omega}\ \bar{k} = -\ddot{\theta}\ \bar{k}$ = angular acceleration. Solving Equation (16) using the vector $\bar{r}_1$ given in Equation (5) gives

$$\bar{a} = \bar{i}\ \left(\ddot{\theta} \sqrt{r_5^2 - x'^2} - (\dot{\theta})^2\ x'\right) - \bar{j}\ \left(\ddot{\theta} x' + (\dot{\theta})^2 \sqrt{r_5^2 - x'^2}\right) \tag{17}$$

To determine $\dot{\theta}$, $\ddot{\theta}$ use equation (11).

$$\begin{aligned} \frac{d\theta}{dt} &= \dot{\theta} = \frac{1}{\left|\bar{r}_5\right|} \frac{ds}{dt} \\ \frac{d^2\theta}{dt^2} &= \ddot{\theta} = \frac{1}{\left|\bar{r}_5\right|} \frac{d^2 s}{dt^2} \end{aligned} \tag{18}$$

One can determine from Figure 5 that:

$$x' = \left|\bar{r}_5\right| \sin \theta \tag{19}$$

For small angular displacements x' will be approximately:

$$x' \approx \left|\bar{r}_5\right| \theta \tag{20}$$

The chord displacement (Δs) will be proportional to the displacement of actuator No. 1 ($\Delta \ell$) for small displacements, i.e.,

$$\Delta s \approx k \, \Delta \ell \tag{21}$$

The value of the constant of proportionality, k, can be determined to be approximately 1.54 as shown in Figure 7[1,2] when the point mass is in the neutral position.

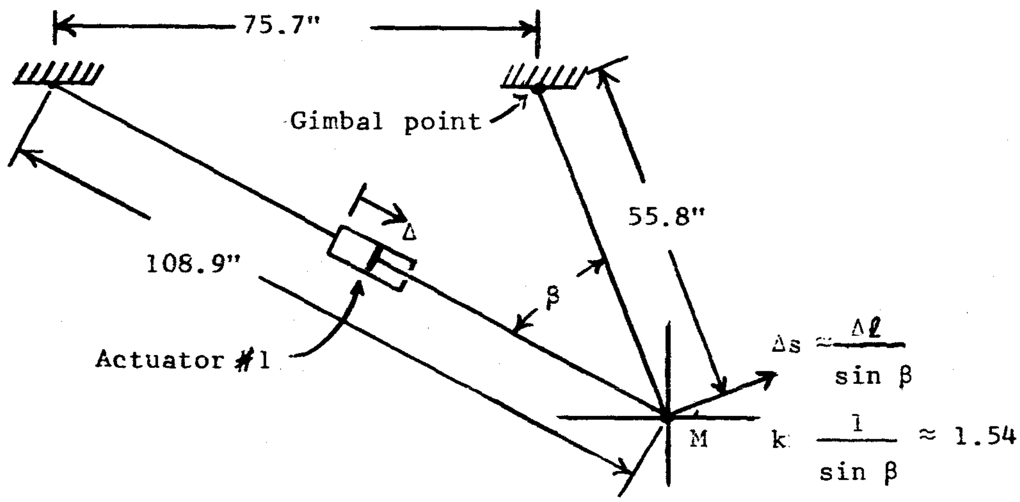


Figure: 7 - Figure Used to Determine the Constant of Proportional k.

Substituting Equation (21) into Equation (18) yields:

$$\dot{\theta} = \frac{k}{\left|\bar{r}_5\right|} \frac{d\ell}{dt}$$
$$\ddot{\theta} = \frac{k}{\left|\bar{r}_5\right|} \frac{d^2\ell}{dt^2} \tag{22}$$

Where $\frac{d\ell}{dt}$ = linear velocity of the actuator and $\frac{d^2\ell}{dt^2}$ is the linear acceleration of the actuator. Using Equations (20) and (22), the acceleration vector (Equation 17) can now be written as:

$$\bar{a} = \bar{i}\left(\frac{k}{|\bar{r}_5|}\frac{d^2\ell}{dt^2}|\bar{r}_5|\sqrt{1-\theta^2} - (\dot{\theta})^2|\bar{r}_5|\theta\right) - \bar{j}\left(\frac{k}{|\bar{r}_5|}\frac{d^2\ell}{dt^2}|\bar{r}_5|\theta + (\dot{\theta})^2|\bar{r}_5|\sqrt{1-\theta^2}\right)$$

and for small angular displacements

$$\bar{a} \approx \bar{i}\left(k\frac{d^2\ell}{dt^2} - (\dot{\theta})^2 |r_5|\theta\right) - \bar{j}\left(k\frac{d^2\ell}{dt^2}\theta + (\dot{\theta})^2 |r_5|\right) \tag{23}$$

From Newton's Law as applied to the kinetics of particles:

$$\text{resultance force} = (\text{mass})\,(\text{acceleration}) \tag{24}$$

It seems reasonable to assume that the maximum acceleration, therefore maximum force, occurs at the instant the actuating signal is applied and before the mass has been displaced any appreciable distance. Therefore $\dot{\theta}$ and θ are very small quantities. With this approximation, equation (24) can be written as:

$$\bar{F} \approx M\left[\bar{i}\left(k\frac{d^2\ell}{dt^2}\right) - \bar{j}\left(k\frac{d^2\ell}{dt^2}\theta\right)\right] \tag{25}$$

It now remains to find the component of force that is transmitted along the axis of the piston of servoactuator No. 2. Recalling that $\bar{r}_2$ is a vector directed from the vehicle tie-point of servoactuator No. 2 to the point mass, it would be more appropriate to use the line of action of a vector drawn from the vehicle tie-point of servoactuator No. 2 to the engine tie-point of servoactuator No. 2 as this gives the true axis of the piston. Since the initial displacements are assumed to be small, the coordinates of this vector with the engine in the neutral position should suffice. Label this vector $\bar{r}_6$. In the x_1, y_1, z_1 coordinate system it is given by [1,2]

$$\bar{r}_6 = 54.4\,\bar{\mu}_{y_1} + 25\,\bar{\mu}_{z_1} \tag{26}$$

and by using the transformations given in Figure 4

$$\bar{r}_6 = 50.3\ \bar{j} + 32.3\ \bar{k} \tag{27}$$

in the x', y', z' coordinate system.

The component of force given by Equation (25) that is transmitted along the axis of the piston of servoactuator No. 2 is

$$|F_{\#2}| = \frac{\bar{F} \cdot \bar{r}_6}{|\bar{r}_6|} . \tag{28}$$

Now

$$|\bar{r}_6| = 59.8''$$

therefore

$$|F_{\#2}| = \frac{M}{59.8}\left[k\ \theta\ \frac{d^2\theta}{dt^2}\right] . \tag{29}$$

The pressure surge developed by this force is $\Delta p = \frac{|F_{\#2}|}{A_p}$ where A_p is the piston area of the servoactuator which is 57 in^2. For a maximum linear acceleration of 12 g's[4] and an engine mass of $\frac{21 \times 10^3}{g}\ \frac{lb}{in/sec^2}$,[1] Equation (29) results in $|F_{\#2}|$ = 6500.0 lbs and

$$\Delta p = \frac{6500 \cdot \theta}{57\ in^2} = 114 \cdot \theta\ lb/in^2. \tag{30}$$

Since θ is less than .1 radians (<5°) which is the maximum angular displacement about the gimbal point from the neutral position, there does not appear to be any danger of a high pressure surge being developed through the mechanical link. The low value does not seem to warrant any further investigation along these lines.

References

(1) Dimensions used are from Fig. 2-15A, Center of Gravity and Inertia Data, Report R-3896-1, 8 June, 1964.

(2) Dimensions used for the actuator tie-points are from Brown Engineering Drawing of "F-1 Engine Simulator Stand." 6-27-65

(3) Beer, F. P. and Johnson, Jr., E. R., Vector Mechanics for Engineers: Statics and Dynamics, McGraw-Hill Book Co., 1962.

(4) From Servoactuator Assembly Specification Number 50M35008,